Solution exercise set 1

1 Experimental Weir equation

First we identify the relevant physical variables:

$$Q = f(H, L, g) \tag{1}$$

$$[Q] = \frac{m^3}{s} \quad [L] = [H] = m \quad [g] = \frac{m}{s^2} \tag{2}$$

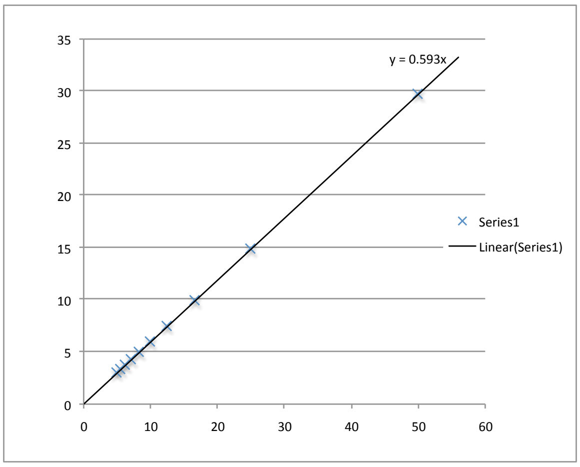
We have therefore two fundamental units: meters and seconds.

Let us choose now H and g as independent variables and make Q dimensionless

$$\Pi = \frac{Q}{H^{5/2} g^{1/2}} = \Psi(\Pi_1) \tag{3}$$

with the dimensionless quantity $\Pi_1 = \frac{L}{H}$. If the result (3) is not straightforward for the reader, it is possible to use the systematic approach and compute the kernel of the dimensional matrix to get the powers 5/2 and 1/2 (left as an exercise).

Plotting Π as a function of Π_1 using the experimental table we get a linear relation $\Pi \approx 0.593 \cdot \Pi_1$:



Reverting to the original variables we get

$$\frac{Q}{H^{5/2} g^{1/2}} = k \frac{L}{H} \Rightarrow Q = k H^{3/2} g^{1/2} L \tag{4}$$

with $k \approx 0.593$.

Usually, this result is written in the following form

$$Q = c\, v_T L H \tag{5}$$

with $v_T = \sqrt{2gH}$ the Torricelli velocity and $c = \frac{k}{\sqrt{2}} \approx 0.42$.

2 "Ideal" Weir equation

The Bernoulli equation states that along a streamline

$$\frac{1}{2}v^2 + \frac{P}{\rho} + gh = Cste. \tag{6}$$

For the ideal Weir, this leads to

$$Q = \int_0^H dh L v(h) = L \int_0^H dh \sqrt{2gh} = L\frac{2}{3}\sqrt{2}H^{3/2}g^{1/2} = \frac{2}{3}v_T H L \tag{7}$$

We have used the fact that the approach velocity is zero and that both the pressures at origin and throughout the nappe are atmospheric. A similar dependence (see equation (4)) was found for the real Weir in the first Problem. The (strong !!) deviation from the actual proportionality constant (0.42 v.s. $\frac{2}{3}$) results from the unrealistic model hypotheses.